

SOUTHWESTERN UNIVERSITY NIGERIA

1. (a) Write an algorithm to find a solution to $f(x) = 0$ given an initial approximation P_0 for the Newton-Raphson method.

(b) Let $f(x) = \frac{1}{2}x^2 + \frac{1}{4}x \sin x + \frac{1}{2} \cos 2x$, $P_0 = \pi/2$. Iterate using Newton's method until an accuracy of 10^{-5} is obtained. Explain why the result seems unusual for Newton's method.

Also, solve the equation with $P_0 = 5\pi$ and $P_0 = 10\pi$. Hence, establish any comparison in these approximations based on the choice of each initial guess.

Instruction: Answer question one and any other two question. **TIME:** 2 1/2 hrs

2. (a) Given a system of a $n \times n$ linear equation of the form

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

1. (a) Write an algorithm to find a solution to $f(x) = 0$ given an initial approximation P_0 for the Newton-Raphson method.

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Also, solve the equation with $P_0 = 5\pi$ and $P_0 = 10\pi$. Hence, establish any comparison in these approximations based on the choice of each initial guess.

(b) Use the Gauss-Seidel iteration method to solve $20x_1 + x_2 + 2x_3 = 17$; $3x_1 + 20x_2 + x_3 = 18$; $2x_1 + 3x_2 + 20x_3 = 25$

2. (a) Given a system of a $n \times n$ linear equation of the form $AX = B$. Compare the rate of convergence of this method with that obtained by Jacobis iterative method.

3. (a) Determine $f(x)$ as a polynomial in x for the following data:

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

Transform the above system of equation into the form $AX = B$ and state any direct methods for solving it.

(b) Use the Gauss-Seidel iteration method to solve $20x_1 + x_2 + 2x_3 = 17$; $3x_1 + 20x_2 + x_3 = 18$; $2x_1 + 3x_2 + 20x_3 = 25$

Compare the rate of convergence of this method with that obtained by Jacobis iterative method.

Hence, compare and state which of the methods gives better approximation.

3. (a) Determine $f(x)$ as a polynomial in x for the following data:

4. (a) Apply the Runge-Kuttas method to the differential equation $y' = -x + y$, $y(0) = 1$

(b) Evaluate the computation of the given initial value problem? $y'' = x^2$, $y(0) = 0$, $y(1) = 1$

(b) Explain the advantage of the Runge-Kuttas method over Eulers method. (i) Trapezoidal rule

5. (a) Obtain an approximate solution of the following boundary value problems (BVP) (ii) Simpsons rule

$$(1 + x^2)y' + 2xy = x^2$$

Hence, compare and state which of the methods gives better approximation.

