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LECTURE NOTE

ON

BUSINESS STATISTICS I (BUA 203/ECO 213)

Lecture I

Introduction to Business Statistics

INTRODUCTION

For a layman, 'Statistics' means numerical information expressed in quantitative terms. This information may relate to objects, subjects, activities, phenomena, or regions of space. As a matter of fact, data have no limits as to their reference, coverage, and scope. At the macro level, these are data on gross national product and shares of agriculture, manufacturing, and services in GDP (Gross Domestic Product).

At the micro level, individual firms, howsoever small or large, produce extensive statistics on their operations. The annual reports of companies contain variety of data on sales, production, expenditure, inventories, capital employed, and other activities.
These data are often field data, collected by employing scientific survey techniques.
Unless regularly updated, such data are the product of a one-time effort and have limited use beyond the situation that may have called for their collection. A student knows statistics more intimately as a subject of study like economics, mathematics, chemistry, physics, and others. It is a discipline, which scientifically deals with data,and is often described as the science of data. In dealing with statistics as data, statistics has developed appropriate methods of collecting, presenting, summarizing, and analysing data, and thus consists of a body of these methods.

MEANING AND DEFINITIONS OF STATISTICS

In the beginning, it may be noted that the word 'statistics' is used rather curiously in two senses plural and singular. In the plural sense, it refers to a set of figures or data. In the singular sense, statistics refers to the whole body of tools that are used to collect data, organise and interpret them and, finally, to draw conclusions from them.

It should be noted that both the aspects of statistics are important if the quantitative data are to serve their purpose. If statistics, as a subject, is inadequate and consists of poor methodology, we could not know the right procedure to extract from the data the information they contain. Similarly, if our data are defective or that they are inadequate or inaccurate, we could not reach the right conclusions even though our subject is well developed.

A.L. Bowley has defined statistics as: (i) statistics is the science of counting, (ii) Statistics may rightly be called the science of averages, and (iii) statistics is the science of measurement of social organism regarded as a whole in all its mani3 festations. *Boddington* defined as: Statistics is the science of estimates and probabilities. Further, *W.I. King* has defined Statistics in a wider context, the science of Statistics is the method of judging collective, natural or social

phenomena from the results obtained by the analysis or enumeration or collection of estimates.

Seligman explored that statistics is a science that deals with the methods of collecting, classifying, presenting, comparing and interpreting numerical data collected to throw some light on any sphere of enquiry. *Spiegal* defines statistics highlighting its role in decision-making particularly under uncertainty, as follows: statistics is concerned with scientific method for collecting, organising, summa rising, presenting and analyzing data as well as drawing valid conclusions and making reasonable decisions on the basis of such analysis. According to *Prof. Horace Secrist*, Statistics is the aggregate of facts, affected to a marked extent by multiplicity of causes, numerically expressed, enumerated or estimated according to reasonable standards of accuracy, collected in a systematic manner for a pre-determined purpose, and placed in relation to each other.

From the above definitions, we can highlight the major characteristics of statistics as follows:
(i) *Statistics are the aggregates of facts*. It means a single figure is not statistics. For example, national income of a country for a single year is not statistics but the same for two or more years is statistics.
(ii) *Statistics are affected by a number of factors.* For example, sale of a product depends on a number of factors such as its price, quality, competition, the income of the consumers, and so on.
(iii) *Statistics must be reasonably accurate.* Wrong figures, if analysed, will lead to erroneous conclusions. Hence, it is necessary that conclusions must be based on accurate figures.

(iv) *Statistics must be collected in a systematic manner.* If data are collected in a haphazard manner, they will not be reliable and will lead to misleading conclusions.

(v) *Collected in a systematic manner for a pre-determined purpose*

(vi) Lastly, Statistics should be placed in relation to each other. If one collects data unrelated to each other, then such data will be confusing and will not lead to any logical conclusions. Data should be comparable over time and over space.

TYPES OF DATA AND DATA SOURCES

Statistical data are the basic raw material of statistics. Data may relate to an activity of our interest, a phenomenon, or a problem situation under study. They derive as a result of the process of measuring, counting and/or observing. Statistical data, therefore, refer to those aspects of a problem situation that can be measured, quantified, counted, or classified. Any object subject phenomenon, or activity that generates data through this process is termed as a variable. In other words, a variableis one that shows a degree of variability when successive measurements are recorded.

In statistics, data are classified into two broad categories: quantitative data and qualitative data. This classification is based on the kind of characteristics that are measured.

Quantitative data are those that can be quantified in definite units of measurement.
These refer to characteristics whose successive measurements yield quantifiable observations. Depending on the nature of the variable observed for measurement, quantitative data can be further categorized as continuous and discrete data.

Obviously, a variable may be a continuous variable or a discrete variable.

(i) Continuous data represent the numerical values of a continuous variable. A continuous variable is the one that can assume any value between any two points on a line segment, thus representing an interval of values. The values are quite precise and close to each other, yet distinguishably different. All characteristics such as weight, length, height, thickness, velocity, temperature, tensile strength, etc., represent continuous variables. Thus, the data recorded on these and similar other characteristics are called continuous data. It may be noted that a continuous variable assumes the finest unit of measurement.

Finest in the sense that it enables measurements to the maximum degree of precision.
(ii) Discrete data are the values assumed by a discrete variable. A discrete variable is the one whose outcomes are measured in fixed numbers. Such data are essentially count data. These are derived from a process of counting, such as the number of items possessing or not possessing a certain characteristic.

The number of customers visiting a departmental store everyday, the incoming flights at an airport, and the defective items in a consignment received for sale, are all examples of discrete data.

Qualitative data refer to qualitative characteristics of a subject or an object. A characteristic is qualitative in nature when its observations are defined and noted in terms of the presence or absence of a certain attribute in discrete numbers. These data are further classified as nominal and rank data.

(i) Nominal data are the outcome of classification into two or more categories of items or units comprising a sample or a population according to some quality characteristic. Classification of students according to sex (as males and females), of workers according to skill (as skilled, semi-skilled, and unskilled), and of employees according to the level of education (as matriculates, undergraduates, and post-graduates), all result into nominal data. Given any such basis of classification, it is always possible to assign each item to a particular class and make a summation of items belonging to each class. The count data so obtained are called nominal data.

(ii) Rank data, on the other hand, are the result of assigning ranks to specify order in terms of the integers 1,2,3, ..., n. Ranks may be assigned according to the level of performance in a test.

a contest, a competition, an interview, or a show. The candidates appearing in an interview, for example, may be assigned ranks in integers ranging from I to n, depending on their performance in the interview. Ranks so assigned can be viewed as the continuous values of a variable involving performance as the quality characteristic.

Data sources could be seen as of two types, viz., secondary and primary. The two can be defined as under:

(i) Secondary data: They already exist in some form: published or unpublished - in an identifiable secondary source. They are, generally, available from published source(s), though not necessarily in the form actually required.

(ii) Primary data: Those data which do not already exist in any form, and thus have to be collected for the first time from the primary source(s). By their very nature, these data require fresh and first-time collection covering the whole population or a sample drawn from it.

TYPES OF STATISTICS

There are two major divisions of statistics such as descriptive statistics and inferential statistics. The term **descriptive statistics** deals with collecting, summarizing, and simplifying data, which are otherwise quite unwieldy and voluminous. It seeks to achieve this in a manner that meaningful conclusions can be readily drawn from the data. Descriptive statistics may thus be seen as comprising methods of bringing out and highlighting the latent characteristics present in a set of numerical data. It not only facilitates an understanding of the data and systematic reporting thereof in a manner; and also makes them amenable to further discussion, analysis, and interpretations.

The first step in any scientific inquiry is to collect data relevant to the problem in hand. When the inquiry relates to physical and/or biological sciences, data collection is normally an integral part of the experiment itself. In fact, the very manner in which an experiment is designed, determines the kind of data it would require and/or generate. The problem of identifying the nature and the kind of the relevant data is thus automatically resolved as soon as the design of experiment is finalized. It is possible in the case of physical sciences. In the case of social sciences, where the required data are often collected through a questionnaire from a number of carefully selected respondents, the problem is not that simply resolved. For one thing, designing the questionnaire itself is a critical initial problem. For another, the number of respondents to be accessed for data collection and the criteria for selecting them has their own implications and importance for the quality of results obtained. Further, the data have been collected, these are assembled, organized, and presented in the form of appropriate tables to make them readable. Wherever needed, figures, diagrams, charts, and graphs are also used for better presentation of the data. A useful tabular and graphic presentation of data will require that the raw data be properly classified in accordance with the objectives of investigation and the relational analysis to be carried out.

A well thought-out and sharp data classification facilitates easy description of the hidden data characteristics by means of a variety of summary measures. These include measures of central tendency, dispersion, skewness, and kurtosis, which constitute the essential scope of descriptive statistics. These form a large part of the subject matter of any basic textbook on the subject, and thus they are being discussed in that order here as well.

Inferential statistics, also known as inductive statistics, goes beyond describing a given problem situation by means of collecting, summarizing, and meaningfully presenting the related data. Instead, it consists of methods that are used for drawing inferences, or making broad generalizations, about a totality of observations on the basis of knowledge about a part of that totality. The totality of observations about which an inference may be drawn, or a generalization made, is called a population or a universe. The part of totality, which is observed for data collection and analysis to gain knowledge about the population, is called a sample.

The desired information about a given population of our interest; may also be collected even by observing all the units comprising the population. This total coverage is called census. Getting the desired value for the population through census is not always feasible and practical for various reasons. Apart from time and money considerations making the census operations prohibitive, observing each individual unit of the population with reference to any data characteristic may at times involve even destructive testing. In such cases, obviously, the only recourse available is to employ the partial or incomplete information gathered through a sample for the purpose. This is precisely what inferential statistics does. Thus, obtaining a particular value from the sample information and using it for drawing an inference about the entire population underlies the subject matter of inferential statistics. Consider a situation in which one is required to know the average body weight of all the college students in a given cosmopolitan city during a certain year. A quick and easy way to do this is to record the weight of only 500 students, from out of a total strength of, say, 10000, or an unknown total strength, take the average, and use this average based on incomplete weight data to represent the average body weight of all the college students. In a different situation, one may have to repeat this exercise for some future year and use the quick estimate of average body weight for a comparison. This may be needed, for example, to decide whether the weight of the college students has undergone a significant change over the years compared.

Inferential statistics helps to evaluate the risks involved in reaching inferences or generalizations about an unknown population on the basis of sample information. For example, an inspection of a sample of five battery cells drawn from a given lot may reveal that all the five cells are in perfectly good condition. This information may be used to conclude that the entire lot is good enough to buy or not.

Since this inference is based on the examination of a sample of limited number of cells, it is equally likely that all the cells in the lot are not in order. It is also possible that all the items that may be included in the sample are unsatisfactory. This may be used to conclude that the entire lot is of unsatisfactory quality, whereas the fact may indeed be otherwise. It may, thus, be

noticed that there is always a risk of an inference about a population being incorrect when based on the knowledge of a limited sample.

The rescue in such situations lies in evaluating such risks. For this, statistics provides the necessary methods. These centres on quantifying in probabilistic term the chances of decisions taken on the basis of sample information being incorrect. This requires an understanding of the what, why, and how of probability and probability distributions to equip ourselves with methods of drawing statistical inferences and estimating the degree of reliability of these inferences.

SCOPE OF STATISTICS

Apart from the methods comprising the scope of descriptive and inferential branches of statistics, statistics also consists of methods of dealing with a few other issues of specific nature. Since these methods are essentially descriptive in nature, they have been discussed here as part of the descriptive statistics. These are mainly concerned with the following:

(i) It often becomes necessary to examine how two paired data sets are related. For example, we may have data on the sales of a product and the expenditure incurred on its advertisement for a specified number of years. Given that sales and advertisement expenditure are related to each other, it is useful to examine the nature of relationship between the two and quantify the degree of that relationship. As this requires use of appropriate statistical methods, these falls under the purview of what we call regression and correlation analysis.

(ii) Situations occur quite often when we require averaging (or totalling) of data on prices and/or quantities expressed in different units of measurement. For example, price of cloth may be quoted per meter of length and that of wheat per kilogram of weight. Since ordinary methods of totalling and averaging do not apply to such price/quantity data, special techniques needed for the purpose are developed under index numbers.

(iii) Many a time, it becomes necessary to examine the past performance of an activity with a view to determining its future behaviour. For example, when engaged in the production of a commodity, monthly product sales are an important measure of evaluating performance. This requires compilation and analysis of relevant sales data over time. The more complex the activity, the more varied the data requirements. For profit maximising and future sales planning, forecast of likely sales growth rate is crucial. This needs careful collection and analysis of past sales data. All such concerns are taken care of under time series analysis.

(iv) Obtaining the most likely future estimates on any aspect(s) relating to a business or economic activity has indeed been engaging the minds of all concerned. This is particularly important when it relates to product sales and demand, which serve the necessary basis of production scheduling and planning. The regression, correlation, and time series analyses together help develop the basic methodology to do the needful. Thus, the study of methods

and techniques of obtaining the likely estimates on business/economic variables comprises the scope of what we do under business forecasting.

Keeping in view the importance of inferential statistics, the scope of statistics may finally be restated as consisting of statistical methods which facilitate decision-- making under conditions of uncertainty. While the term statistical methods is often used to cover the subject of statistics as a whole, in particular it refers to methods by which statistical data are analysed, interpreted, and the inferences drawn for decision-making.

Though generic in nature and versatile in their applications, statistical methods have come to be widely used, especially in all matters concerning business and economics. These are also being increasingly used in biology, medicine, agriculture, psychology, and education. The scope of application of these methods has started opening and expanding in a number of social science disciplines as well. Even a political scientist finds them of increasing relevance for examining the political behaviour and it is, of course, no surprise to find even historians statistical data, for history is essentially past data presented in certain actual format.

IMPORTANCE OF STATISTICS IN BUSINESS

There are three major functions in any business enterprise in which the statistical methods are useful. These are as follows:

(i) The planning of operations: This may relate to either special projects or to the recurring activities of a firm over a specified period.
(ii) The setting up of standards: This may relate to the size of employment, volume of sales, fixation of quality norms for the manufactured product, norms for the daily output, and so forth.
(iii) The function of control: This involves comparison of actual production achieved against the norm or target set earlier. In case the production has fallen short of the target, it gives remedial measures so that such a deficiency does not occur again.

A worth noting point is that although these three functions-planning of operations, setting standards, and control-are separate, but in practice they are very much interrelated.

Different authors have highlighted the importance of Statistics in business. For instance, Croxton and Cowden give numerous uses of Statistics in business such as project planning, budgetary planning and control, inventory planning and control, quality control, marketing, production and personnel administration. Within these also they have specified certain areas where Statistics is very relevant. Another author, Irwing W. Burr, dealing with the place of statistics in an industrial organisation, specifies a number of areas where statistics is extremely useful. These are: customer wants and market research, development design and specification, purchasing, production, inspection, packaging and shipping, sales and complaints, inventory and maintenance, costs, management control, industrial engineering and research.

Statistical problems arising in the course of business operations are multitudinous. As such, one may do no more than highlight some of the more important ones to emphasis the relevance of statistics to the business world. In the sphere of production, for example, statistics can be useful in various ways.

Statistical quality control methods are used to ensure the production of quality goods. Identifying and rejecting defective or substandard goods achieve this. The sale targets can be fixed on the basis of sale forecasts, which are done by using varying methods of forecasting. Analysis of sales affected against the targets set earlier would indicate the deficiency in achievement, which may be on account of several causes: (i) targets were too high and unrealistic (ii) salesmen's performance has been poor (iii) emergence of increase in competition (iv) poor quality of company's product, and so on. These factors can be further investigated.

Another sphere in business where statistical methods can be used is personnel management. Here, one is concerned with the fixation of wage rates, incentive norms and performance appraisal of individual employee. The concept of productivity is very relevant here. On the basis of measurement of productivity, the productivity bonus is awarded to the workers. Comparisons of wages and productivity are undertaken in order to ensure increases in industrial productivity.

Statistical methods could also be used to ascertain the efficacy of a certain product, say, medicine. For example, a pharmaceutical company has developed a new medicine in the treatment of bronchial asthma. Before launching it on commercial basis, it wants to ascertain the effectiveness of this medicine. It undertakes an experimentation involving the formation of two comparable groups of asthma patients. One group is given this new medicine for a specified period and the other one is treated with the usual medicines. Records are maintained for the two groups for the specified period. This record is then analysed to ascertain if there is any significant difference in the recovery of the two groups. If the difference is really significant statistically, the new medicine is commercially launched.

LIMITATIONS OF STATISTICS

Statistics has a number of limitations, pertinent among them are as follows:
(i) There are certain phenomena or concepts where statistics cannot be used. This is because these phenomena or concepts are not amenable to measurement. For example, beauty, intelligence, courage cannot be quantified. Statistics has no place in all such cases where quantification is not possible.

(ii) Statistics reveal the average behaviour, the normal or the general trend. An application of the 'average' concept if applied to an individual or a particular situation may lead to a wrong conclusion and sometimes may be disastrous. For example, one may be misguided when told that the average depth of a river from one bank to the other is four feet, when there may be some points in between where its depth is far more than four feet. On this understanding, one may enter those points having greater depth, which may be hazardous.

(iii) Since statistics are collected for a particular purpose, such data may not be relevant or useful in other situations or cases. For example, secondary data (i.e., data originally collected by someone else) may not be useful for the other person.

(iv) Statistics are not 100 per cent precise as is Mathematics or Accountancy. Those who use statistics should be aware of this limitation.

(v) In statistical surveys, sampling is generally used as it is not physically possible to cover all the units or elements comprising the universe. The results may not be appropriate as far as the universe is concerned. Moreover, different surveys based on the same size of sample but different sample units may yield different results.

(vi) At times, association or relationship between two or more variables is studied in statistics, but such a relationship does not indicate cause and effect' relationship. It simply shows the similarity or dissimilarity in the movement of the two variables. In such cases, it is the user who has to interpret the results carefully, pointing out the type of relationship obtained.

(vii) A major limitation of statistics is that it does not reveal all pertaining to a certain phenomenon. There is some background information that statistics does not cover. Similarly, there are some other aspects related to the problem on hand, which are also not covered. The user of Statistics has to be well informed and should interpret Statistics keeping in mind all other aspects having relevance on the given problem.

Apart from the limitations of statistics mentioned above, there are misuses of it. Manypeople, knowingly or unknowingly, use statistical data in wrong manner. Let us see what the main misuses of statistics are so that the same could be avoided when one has to use statistical data. The misuse of Statistics may take several forms some of which are explained below:

(i) Sources of data not given: At times, the source of data is not given. In the absence of the source, the reader does not know how far the data are reliable. Further, if he wants to refer to the original source, he is unable to do so.

(ii) Defective data: Another misuse is that sometimes one gives defective data. This may be done knowingly in order to defend one's position or to prove a particular point. This apart, the definition used to denote a certain phenomenon may be defective. For example, in case of data relating to unemployed persons, the definition may include even those who are employed, though partially. The question here is how far it is justified to include partially employed persons amongst unemployed ones.

(iii) Unrepresentative sample: In statistics, several times one has to conduct a survey, which necessitates to choose a sample from the given population or universe. The sample may turn out to be unrepresentative of the universe. One may choose a sample just on the basis of convenience. He may collect the desired information from either his friends or nearby

respondents in his neighbourhood even though such respondents do not constitute a representative sample.

(iv) Inadequate sample: Earlier, we have seen that a sample that is unrepresentative of the universe is a major misuse of statistics. This apart, at times one may conduct a survey based on an extremely inadequate sample. For example, in a city we may find that there are 1, 00,000 households. When we have to conduct a household survey, we may take a sample of merely households comprising only 0.1 per cent of the universe. A survey based on such a small sample may not yield right information.

(v) Unfair Comparisons: An important misuse of statistics is making unfair comparisons from the data collected. For instance, one may construct an index of production choosing the base year where the production was much less. Then he may compare the subsequent year's production from this low base.

Such a comparison will undoubtedly give a rosy picture of the production though in reality it is not so. Another source of unfair comparisons could be when one makes absolute comparisons instead of relative ones. An absolute comparison of two figures, say, of production or export, may show a good increase, but in relative terms it may turnout to be very negligible. Another example of unfair comparison is when the population in two cities is different, but a comparison of overall death rates and deaths by a particular disease is attempted. Such a comparison is wrong. Likewise, when data are not properly classified or when changes in the composition of population in the two years are not taken into consideration, comparisons of such data would be unfair as they would lead to misleading conclusions.

(vi) Unwanted conclusions: Another misuse of statistics may be on account of unwarranted conclusions. This may be as a result of making false assumptions. For example, while making projections of population in the next five years, one may assume a lower rate of growth though the past two years indicate otherwise. Sometimes one may not be sure about the changes in business environment in the near future. In such a case, one may use an assumption that may turn out to be wrong. Another source of unwarranted conclusion may be the use of wrong average. Suppose in a series there are extreme values, one is too high while the other is too low, such as 800 and 50. The use of an arithmetic average in such a case may give a wrong idea. Instead, harmonic mean would be proper in such a case.

(vii) Confusion of correlation and causation: In statistics, several times one has to examine the relationship between two variables. A close relationship between the two variables may not establish a cause-and-effect-relationship in the sense that one variable is the cause and the other is the effect. It should be taken as something that measures degree of association rather than try to find out causal relationship.

Lecture II

An Overview of Central Tendency

INTRODUCTION

The description of statistical data may be quite elaborate or quite brief depending on two factors: the nature of data and the purpose for which the same data have been collected. While describing data statistically or verbally, one must ensure that the description is neither too brief nor too lengthy. The measures of central tendency enable us to compare two or more distributions pertaining to the same time period or within the same distribution over time. For example, the average consumption of tea in two different territories for the same period or in a territory for two years, say, 2003 and 2004, can be attempted by means of an average.

AN OVERVIEW OF CENTRAL TENDENCY

ARITHMETIC MEAN

Adding all the observations and dividing the sum by the number of observations results the arithmetic mean. Suppose we have the following observations: 10, 15, 30, 7, 42, 79 and 83
These are seven observations. Symbolically, the arithmetic mean, also called simply *mean* is

$$x = \Sigma x/n,$$

where x is simple mean.

$$= \frac{10 + 15 + 30 + 7 + 42 + 79 + 83}{7}$$

$$= \frac{266}{7}$$

$$= 38$$

It may be noted that the Greek letter μ is used to denote the mean of the population and n to denote the total number of observations in a population. Thus the population mean $\mu = \Sigma x/n.$ The formula given above is the basic formula that forms the definition of arithmetic mean and is used in case of ungrouped data where weights are not involved.

UNGROUPED DATA-WEIGHTED AVERAGE

In case of ungrouped data where weights are involved, our approach for calculating arithmetic mean will be different from the one used earlier.

Example: Suppose a student has secured the following marks in three tests:

Mid-term test	30
Laboratory	25
Final	20

The simple arithmetic mean will be $\frac{30 + 25 + 20}{3}$

$= 25$

However, this will be wrong if the three tests carry different weights on the basis of their relative importance. Assuming that the weights assigned to the three tests are:

Mid-term test	2 points
Laboratory	3 points
Final	5 points

Solution: On the basis of this information, we can now calculate a weighted mean as shown below:

Table 2.1: Calculation of a Weighted Mean

Type of Test	Before Weight (w)	Marks (x)	(wx)
Mid-term	2	30	60
Laboratory	3	25	75
Final	5	20	100
Total	10		235

$$\bar{x} = \frac{\sum wx}{\sum w} = \frac{w_1x_1 + w_2x_2 + w_3x_3}{w_1 + w_2 + w_3}$$

$$= \frac{60 + 75 + 100}{2 + 3 + 5} = 23.5 \text{ marks}$$

It will be seen that weighted mean gives a more realistic picture than the simple or unweighted mean.

GROUPED DATA-ARITHMETIC MEAN

For grouped data, arithmetic mean may be calculated by applying any of the following methods:

(i) Direct method, (ii) Short-cut method, (iii) Step-deviation method

In the case of direct method, the formula $x = \Sigma fm/n$ is used. Here m is mid-point of various classes, f is the frequency of each class and n is the total number of frequencies. The calculation of arithmetic mean by the direct method is shown below:

Example: The following table gives the marks of 58 students in Statistics. Calculate the average marks of this group.

Marks	*No. of Students*
0-10	4
10-20	8
20-30	11
30-40	15
40-50	12
50-60	6
60-70	2
Total	58

Solution:

Table: Calculation of Arithmetic Mean by Direct Method

Marks	Mid-Points m	No. of Students f	fm
0-10	5	4	20
10-20	15	8	120
21-30	25	11	275
30-40	35	15	525
40-50	45	12	540
50-60	55	6	330
60-70	65	2	130
			Σfm = 1940

where,

$$\bar{x} = \frac{\sum fm}{n} = \frac{1940}{58} = 33.45 \text{ marks or 33 marks approximately}$$

It may be noted that the mid-point of each class is taken as a good approximation of the true mean of the class. This is based on the assumption that the values are distributed fairly evenly throughout the interval. When large numbers of frequency occur, this assumption is usually accepted.

In the case of short-cut method, the concept of arbitrary mean is followed. The formula for calculation of the arithmetic mean by the short-cut method is given below:

$$\bar{x} = A + \frac{\sum fd}{n}$$

where

A = arbitrary or assumed mean
f = frequency
d = deviation from the arbitrary or assumed mean

When the values are extremely large and/or in fractions, the use of the direct method would be very cumbersome. In such cases, the short-cut method is preferable. This is because the calculation work in the short-cut method is considerably reduced particularly for calculation of the product of values and their respective frequencies. However, when calculations are not made manually but by a machine calculator, it may not be necessary to resort to the short-cut method, as the use of the direct method may not pose any problem.

As can be seen from the formula used in the short-cut method, an arbitrary or assumed mean is used. The second term in the *formula (*Σ*fd* ÷ n) is the correction factor for the difference between the actual mean and the assumed mean. If the assumed mean turns out to be equal to the actual mean, (Σ*fd* ÷ n) will be zero. The use of the short-cut method is based on the principle that the total of deviations taken from an actual mean is equal to zero. As such, the deviations taken from any other figure will depend on how the assumed mean is related to the actual mean. While one may choose any value as assumed mean, it would be proper to avoid extreme values, that is, too small or too high to simplify calculations. A value apparently close to the arithmetic mean should be chosen.

For the figures given earlier pertaining to marks obtained by 58 students, we calculate the average marks by using the short-cut method.

Example:

Table 2.4: Calculation of Arithmetic Mean by Short-cut Method

Marks	Mid-Points m	f	d	fd
0-10	5	4	-30	-120
10-20	15	8	-20	-160
21-30	25	11	-10	-110
30-40	35	15	0	0
40-50	45	12	10	120
50-60	55	6	20	120
60-70	65	2	30	60
				Σfd = -90

It may be noted that we have taken arbitrary mean as 35 and deviations from midpoints. In other words, the arbitrary mean has been subtracted from each value of mid-point and the resultant figure is shown in column *d.*

$$\bar{x} = A + \frac{\sum fd}{n}$$
$$= 35 + \frac{-90}{58}$$

= 35 - 1.55 = 33.45 or 33 marks approximately.

Now we take up the calculation of arithmetic mean for the same set of data using the step-deviation method. This is shown in Table 2.5.

Table 2.5: Calculation of Arithmetic Mean by Step-deviation Method

Marks	Mid-Points m	f	d	d'=d/10	fd
0-10	5	4	-30	-3	-12
10-20	15	8	-20	-2	-16
21-30	25	11	-10	-1	-11
30-40	35	15	0	0	0
40-50	45	12	10	1	12
50-60	55	6	20	2	12
60-70	65	2	30	3	6
					Σfd = -9

$$\bar{x} = A + \frac{\sum fd'}{n} xC$$
$$= 35 + \frac{-9x10}{58}$$

= 33.45 or 33 marks approximately.

It will be seen that the answer in each of the three cases is the same. The step deviation method is the most convenient on account of simplified calculations. It may also be noted that if we select a different arbitrary mean and recalculate deviations from that figure, we would get the same answer.

Now that we have learnt how the arithmetic mean can be calculated by using different methods, we are in a position to handle any problem where calculation of the arithmetic mean is involved.

Example: The mean of the following frequency distribution was found to be 1.46.

No. of Accidents	*No. of Days (frequency)*
0	46
1	?
2	?
3	25
4	10
5	5
Total	200 days

Calculate the missing frequencies.

Solution:

Here we are given the total number of frequencies and the arithmetic mean. We have to determine the two frequencies that are missing. Let us assume that the frequency against 1 accident is x and against 2 accidents is y. If we can establish two simultaneous equations, then we can easily find the values of X and *Y.*

$$\text{Mean} = \frac{(0.46) + (1 \,.\, x) + (2 \,.\, y) + (3 \,.\, 25) + (4 \,.\, l0) + (5 \,.\, 5)}{200}$$

$$1.46 = \frac{x + 2y + 140}{200}$$

$x + 2y + 140 = (200)\ (1.46)$

$x + 2y = 152$

$x + y = 200 - \{46+25 + 10+5\}$

$x + y = 200 - 86$

$x + y = 114$

Now subtracting equation (ii) from equation (i), we get

$$\begin{array}{l} x + 2y = 152 \\ x + \ \ y = 114 \\ \hline \ \ \ \ y = 38 \end{array}$$

Substituting the value of $y = 38$ in equation (ii) above, $x + 38 = 114$

Therefore, $x = 114 - 38 = 76$

Hence, the missing frequencies are:

Against accident 1: 76

Against accident 2: 38

CHARACTERISTICS OF THE ARITHMETIC MEAN

Some of the important characteristics of the arithmetic mean are:

1. The sum of the deviations of the individual items from the arithmetic mean is always zero. This means I: $(x - \bar{x}) = 0$, where x is the value of an item and $\bar{x}$ is the arithmetic mean. Since the sum of the deviations in the positive direction is equal to the sum of the deviations in the negative direction, the arithmetic mean is regarded as a measure of central tendency.

2. The sum of the squared deviations of the individual items from the arithmetic mean is always minimum. In other words, the sum of the squared deviations taken from any value other than the arithmetic mean will be higher.

3. As the arithmetic mean is based on all the items in a series, a change in the value of any item will lead to a change in the value of the arithmetic mean.

4. In the case of highly skewed distribution, the arithmetic mean may get distorted on account of a few items with extreme values. In such a case, it may cease to be the representative characteristic of the distribution.

MEDIAN

Median is defined as the value of the middle item (or the mean of the values of the two middle items) when the data are arranged in an ascending or descending order of magnitude. Thus, in an ungrouped frequency distribution if the n values are arranged in ascending or descending order of magnitude, the median is the middle value if n is odd. When n is even, the median is the mean of the two middle values.

Suppose we have the following series:

15, 19,21,7, 10,33,25,18 and 5

We have to first arrange it in either ascending or descending order. These figures are arranged in an ascending order as follows:

5,7,10,15,18,19,21,25,33

Now as the series consists of odd number of items, to find out the value of the middle item, we use the formula

Where $\frac{n+1}{2}$

Where *n* is the number of items. In this case, *n* is 9, as such $\frac{n+1}{2}$ = 5, that is, the size of the 5th item is the median. This happens to be 18.

Suppose the series consists of one more items 23. We may, therefore, have to include 23 in the above series at an appropriate place, that is, between 21 and 25. Thus, the series is now 5, 7, 10, 15, 18, 19, and 21,23,25,33. Applying the above formula, the median is the size of 5.5th item. Here, we have to take the average of the values of 5th and 6th item. This means an average of 18 and 19, which gives the median as 18.5. It may be noted that the formula $\frac{n+1}{2}$ itself is not the formula for the median; it merely indicates the position of the median, namely, the number of items we have to count until we arrive at the item whose value is the median. In the case of the even number of items in the series, we identify the two items whose values have to be averaged to obtain the median. In the case of a grouped series, the median is calculated by linear interpolation with the help of the following formula:

$$M = l_1 \frac{l_2}{f} + l_1(m - c)$$

where

M = the median
l$_1$ = the lower limit of the class in which the median lies
1$_2$ = the upper limit of the class in which the median lies
f = the frequency of the class in which the median lies
m = the middle item or *(n* + 1)/2th, where *n* stands for total number of items
c = the cumulative frequency of the class preceding the one in which the median lies

Example:

Monthly Wages (N)	*No. of Workers*
800-1,000	18
1,000-1,200	25
1,200-1,400	30
1,400-1,600	34

1,600-1,800	26
1,800-2,000	10
Total	143

In order to calculate median in this case, we have to first provide cumulative frequency to the table. Thus, the table with the cumulative frequency is written as:

Monthly Wages	Frequency	Cumm. Frequency
800-1,000	18	18
1,000-1,200	25	43
1,200-1,400	30	73
1,400-1,600	34	107
1,600-1,800	26	133
1,800-2,000	10	143

$$M = l_1 \frac{l_2}{f} + l_1(m-c)$$

$$M = \frac{n+1}{2} = \frac{143+1}{2} = 72$$

It means median lies in the class-interval ₦1,200 - 1,400.

Now, M = $1200 + \frac{1400-1200}{30}(72\ 43)$

$= 1200 + \frac{200}{30}(29)$

= ₦1393.3

At this stage, let us introduce two other concepts viz. quartile and decile. To understand these, we should first know that the median belongs to a general class of statistical descriptions *called fractiles.* A fractile is a value below that lays a given fraction of a set of data. In the case of the median, this fraction is one-half (1/2).

Likewise, a quartile has a fraction one-fourth (1/4). The three quartiles Q_1, Q_2 and Q_3 are such that 25 percent of the data fall below Q_1, 25 percent fall between Q_1 and Q_2, 25 percent fall between Q_2 and Q_3 and 25 percent fall above Q_3 It will be seen that Q_2 is the median. We can use the above formula for the calculation of quartiles as well.

The only difference will be in the value of m. Let us calculate both Q_1 and Q_3 in respect of the table given in Example.

$$Q_1 = l_1 \frac{l_2 - l_1}{f}(m - c)$$

Hence, m will be $= \frac{n+1}{4} = \frac{143+1}{4} = 36$

$$Q_1 = 1000 + \frac{1200 - 1000}{25}(36 - 18)$$

$$= 1000 + \frac{200}{25}(18)$$

= ₦1,144

In the case of Q_3, m will be 3 $= \frac{n+1}{4} = \frac{3x144}{4} = 108$

$$Q_3 = 1600 + \frac{1800 - 1600}{26}(108 - 107)$$

$$= 1600 + \frac{200}{26}(1)$$

= ₦ 1,607.7 approx.

In the same manner, we can calculate deciles (where the series is divided into 10 parts) and percentiles (where the series is divided into 100 parts). It may be noted that unlike arithmetic mean, median is not affected at all by extreme values, as it is a positional average. As such, median is particularly very useful when a distribution happens to be skewed. Another point that goes in favour of median is that it can be computed when a distribution has open-end classes. Yet, another merit of median is that when a distribution contains qualitative data, it is the only average that can be used. No other average is suitable in case of such a distribution. Let us take a couple of examples to illustrate what has been said in favour of median.

Example: Calculate the most suitable average for the following data:

Size of the Item	Below 50	50-100	100-150	150-200	200 and above
Frequency	*15*	*20*	*36*	*40*	*10*

Solution: Since the data have two open-end classes-one in the beginning (below 50) and the other at the end (200 and above), median should be the right choice as a measure of central tendency.

Table 2.6: Computation of Median

Size of Item	*Frequency*	*Cumulative Frequency*
Below 50	15	15
50-100	20	35
100-150	36	71
150-200	40	111
200 and above	10	121

Median is the size of $\frac{n+1}{2}$ th item

$\frac{121+1}{2}$ = 61st item

Now, 61st item lies in the 100-150 class

Median = $Q_1 = l_1 \frac{l_2 - l_1}{f}(m-c)$

$= 100 + \frac{150-100}{36}(61-35)$

= 100 + 36.11 = 136.11 approx.

Example: The following data give the savings bank accounts balances of nine sample households selected in a survey. The figures are in naira.

745 2,000 1,500 68,000 461 549 3750 1800 4795

(a) Find the mean and the median for these data; (b) Do these data contain an outlier? If so, exclude this value and recalculate the mean and median. Which of these summary measures has a greater change when an outlier is dropped?; (c) Which of these two summary measures is more appropriate for this series?

Solution:

Mean = $\frac{₦745 + 2,000 + 1,500 + 68,000 + 461 + 549 + 3,750 + 1,800 + 4,795}{9}$

= $\frac{83,600}{9} = N9,289$

Median = size of $\frac{n+1}{2}$ th item

= $\frac{9+1}{2}$ = 5th item

Arranging the data in an ascending order, we find that the median is ₦1,800.

(b) An item of Rs 68,000 is excessively high. Such a figure is called an 'outlier'. We exclude this figure and recalculate both the mean and the median.

$$\text{Mean} = \frac{83{,}600 - 68{,}000}{8}$$

$$= \frac{15{,}600}{8} = 1{,}950$$

Median = Size of $\frac{n+1}{2}$ th item

$$\frac{8+1}{2} = 4.5^{th} \text{ item}$$

$$\frac{1500 + 1800}{2} = 1{,}650$$

It will be seen that the mean shows a far greater change than the median when the outlier is dropped from the calculations.

(c) As far as these data are concerned, the median will be a more appropriate measure than the mean.

Further, we can determine the median graphically as follows:

Example: Suppose we are given the following series:

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	6	12	22	37	17	8	5

We are asked to draw both types of O'give from these data and to determine the median.

Solution:
First of all, we transform the given data into two cumulative frequency distributions, one based on 'less than' and another on 'more than' methods.

Table A

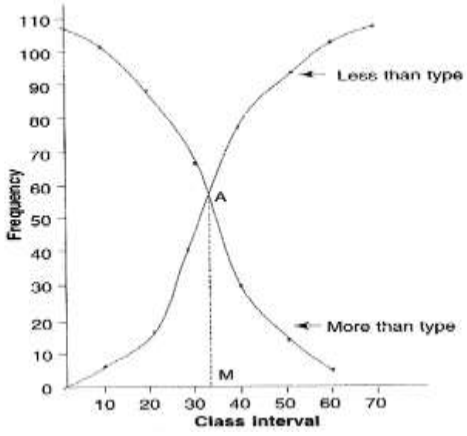
	Frequency
Less than 10	6
Less than 20	18
Less than 30	40
Less than 40	77

Less than 50	94
Less than 60	102
Less than 70	107

Table B

	Frequency
More than 0	107
More than 10	101
More than 20	89
More than 30	67
More than 40	30
More than 50	13
More than 60	5

It may be noted that the point of intersection of the two O'gives gives the value of the median.



From this point of intersection A, we draw a straight line to meet the X-axis at M. Thus, from the point of origin to the point at M gives the value of the median, which comes to 34, approximately. If we calculate the median by applying the formula, then the answer comes to 33.8, or 34, approximately. It may be pointed out that even a single ogive can be used to determine the median. As we have determined the median graphically, so also we can find the values of quartiles, deciles or percentiles graphically. For example, to determine we have to take size of $\{3(n + 1)\} /4 = 81_{st}$ item. From this point on the Y-axis, we can draw a perpendicular to meet the 'less than' ogive from which another straight line is to be drawn to meet the X-axis. This point will give us the value of the upper quartile. In the same manner, other values of Q_1 and deciles and percentiles can be determined.

CHARACTERISTICS OF THE MEDIAN

1. Unlike the arithmetic mean, the median can be computed from open-ended distributions. This is because it is located in the median class-interval, which would not be an open-ended class.
2. The median can also be determined graphically whereas the arithmetic mean cannot be ascertained in this manner.
3. As it is not influenced by the extreme values, it is preferred in case of a distribution having extreme values.
4. In case of the qualitative data where the items are not counted or measured but are scored or ranked, it is the most appropriate measure of central tendency.

MODE

The mode is another measure of central tendency. It is the value at the point around which the items are most heavily concentrated. As an example, consider the following series:

8, 9, 11, 15, 16, 12, 15,3, 7, 15 37

There are ten observations in the series wherein the figure 15 occurs maximum number of times three. The mode is therefore 15. The series given above is a discrete series; as such, the variable cannot be in fraction. If the series were continuous, we could say that the mode is approximately 15, without further computation.

In the case of grouped data, mode is determined by the following formula:

$$\text{Mode} = l_1 \frac{f_1 - f_0}{(f_1 - f_0) + (f_1 - f_2)} xi$$

where,

l_1 = the lower value of the class in which the mode lies
f_l = the frequency of the class in which the mode lies
f_o = the frequency of the class preceding the modal class
f_2 = the frequency of the class succeeding the modal class
i = the class-interval of the modal class

While applying the above formula, we should ensure that the class-intervals are uniform throughout. If the class-intervals are not uniform, then they should be made uniform on the assumption that the frequencies are evenly distributed throughout the class. In the case of inequal class-intervals, the application of the above formula will give misleading results.

Example: Let us take the following frequency distribution:

Class intervals (1)	*Frequency (2)*
30-40	4
40-50	6
50-60	8
60-70	12
70-80	9
80-90	7
90-100	4

We have to calculate the mode in respect of this series.

Solution: We can see from Column (2) of the table that the maximum frequency of 12 lies in the class-interval of 60-70. This suggests that the mode lies in this class interval. Applying the formula given earlier, we get:

$$\text{Mode} = 60 + \frac{12-8}{(12-8)+12-9)} x10$$

$$= 60\frac{4}{4+3} x10$$

= 65.7 approx.

In several cases, just by inspection one can identify the class-interval in which the mode lies. One should see which the highest frequency is and then identify to which class-interval this frequency belongs. Having done this, the formula given for calculating the mode in a grouped frequency distribution can be applied.

At times, it is not possible to identify by inspection the class where the mode lies. In such cases, it becomes necessary to use the method of grouping. This method consists of two parts:

(i) **Preparation of a grouping table:** A grouping table has six columns, the first column showing the frequencies as given in the problem. Column 2 shows frequencies grouped in two's, starting from the top. Leaving the first frequency, column 3 shows frequencies grouped in two's.

Column 4 shows the frequencies of the first three items, then second to fourth item and so on.
Column 5 leaves the first frequency and groups the remaining items in three's.
Column 6 leaves the first two frequencies and then groups the remaining in three's. Now, the maximum total in each column is marked and shown either in a circle or in a bold type.

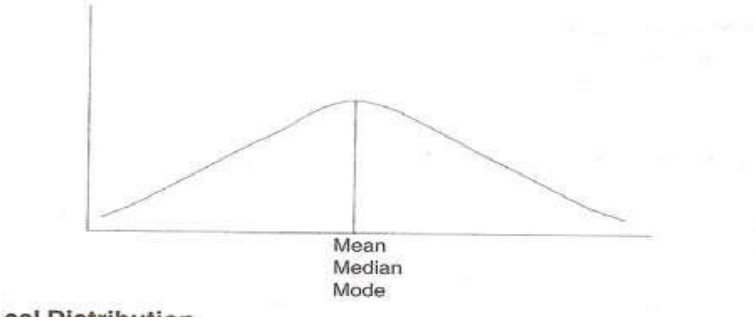
(ii) **Preparation of an analysis table**: After having prepared a grouping table, an analysis table is prepared. On the left-hand side, provide the first column for column numbers and on the right-

hand side the different possible values of mode. The highest values marked in the grouping table are shown here by a bar or by simply entering 1 in the relevant cell corresponding to the values they represent. The last row of this table will show the number of times a particular value has occurred in the grouping table. The highest value in the analysis table will indicate the class-interval in which the mode lies. The procedure of preparing both the grouping and analysis tables to locate the modal class will be clear by taking an example.

RELATIONSHIPS OF THE MEAN, MEDIAN AND MODE

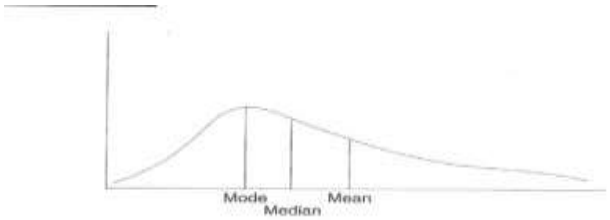
Having discussed mean, median and mode, we now turn to the relationship amongst these three measures of central tendency. We shall discuss the relationship assuming that there is a unimodal frequency distribution.

(i) When a distribution is symmetrical, the mean, median and mode are the same, as is shown below in the following figure.
In case, a distribution is skewed to the right, then mean> median> mode.

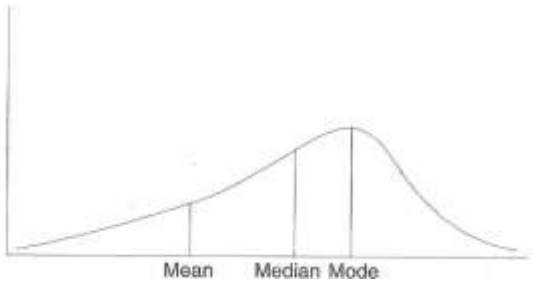


Generally, income distribution is skewed to the right where a large number of families have relatively low income and a small number of families have extremely high income. In such a case, the mean is pulled up by the extreme high incomes and the relation among these three measures is as shown in Fig. 6.3. Here, we find that mean> median> mode.

(ii) When a distribution is skewed to the left, then mode> median> mean. This is because here mean is pulled down below the median by extremely low values. This is shown as in the figure.



(iii) Given the mean and median of a unimodal distribution, we can determine whether it is skewed to the right or left. When mean> median, it is skewed to the right; when median> mean, it is skewed to the left. It may be noted that the median is always in the middle between mean and mode.



THE BEST MEASURE OF CENTRAL TENDENCY

At this stage, one may ask as to which of these three measures of central tendency the best is. There is no simple answer to this question. It is because these three measures are based upon different concepts. The arithmetic mean is the sum of the values divided by the total number of observations in the series. The median is the value of the middle observation that divides the series into two equal parts. Mode is the value around which the observations tend to concentrate. As such, the use of a particular measure will largely depend on the purpose of the study and the nature of the data;

For example, when we are interested in knowing the consumers preferences for different brands of television sets or different kinds of advertising, the choice should go in favour of mode. The use of mean and median would not be proper. However, the median can sometimes be used in the case of qualitative data when such data can be arranged in an ascending or descending order. Let us take another example.

Suppose we invite applications for a certain vacancy in our company. A large number of candidates apply for that post. We are now interested to know as to which age or age group has the largest concentration of applicants. Here, obviously the mode will be the most appropriate choice. The arithmetic mean may not be appropriate as it may be influenced by some extreme values. However, the mean happens to be the most commonly used measure of central tendency as will be evident from the discussion in the subsequent chapters.

GEOMETRIC MEAN

Apart from the three measures of central tendency as discussed above, there are two other means that are used sometimes in business and economics. These are the geometric mean and the harmonic mean. The geometric mean is more important than the harmonic mean. We

discuss below both these means. First, we take up the geometric mean. Geometric mean is defined at the *nth* root of the product of *n* observations of a distribution.

Symbolically, GM = $\sqrt[n]{x_1}....x_2.....x_n$ If we have only two observations, say, 4 and 16 then GM = $\sqrt{4x16}$ = 64 = 8. Similarly, if there are three observations, then we have to calculate the cube root of the product of these three observations; and so on. When the number of items is large, it becomes extremely difficult to multiply the numbers and to calculate the root. To simplify calculations, logarithms are used.

Example: If we have to find out the geometric mean of 2, 4 and 8, then we find

$$\text{Log GM} = \frac{\sum \log x_i}{n}$$

$$= \frac{Log2 + Log4 + Log8}{3}$$

$$= \frac{0.3010 + 0.6021 + 0.9031}{3}$$

$$= \frac{1.8062}{3}$$

$$= 0.60206$$

$$\text{GM} = \text{Antilog } 0.60206$$

$$= 4$$

When the data are given in the form of a frequency distribution, then the geometric mean can be obtained by the formula:

$$\text{Log GM} = \frac{f_1.\log x_1 + f_2.\log x_2 + ... + f_n.\log x_n}{f_1 + f_2 + ... + f_n}$$

$$= \frac{\sum f.\log x}{f_1 + f_2 + ... + f_n}$$

Then, GM = Antilog *n*

The geometric mean is most suitable in the following three cases:
1. Averaging rates of change.
2. The compound interest formula.
3. Discounting, capitalization.

ADVANTAGES OF G. M.

1. Geometric mean is based on each and every observation in the data set.
2. It is rigidly defined.
3. It is more suitable while averaging ratios and percentages as also in calculating growth rates.
4. As compared to the arithmetic mean, it gives more weight to small values and less weight to large values. As a result of this characteristic of the geometric mean, it is generally less than the arithmetic mean. At times it may be equal to the arithmetic mean.
5. It is capable of algebraic manipulation. If the geometric mean has two or more series is known along with their respective frequencies. Then a combined geometric mean can be calculated by using the logarithms.

LIMITATIONS OF G.M.

1. As compared to the arithmetic mean, geometric mean is difficult to understand.
2. Both computation of the geometric mean and its interpretation are rather difficult.
3. When there is a negative item in a series or one or more observations have zero value, then the geometric mean cannot be calculated. In view of the limitations mentioned above, the geometric mean is not frequently used.

HARMONIC MEAN

The harmonic mean is defined as the reciprocal of the arithmetic mean of the reciprocals of individual observations. Symbolically,

$$\text{HM} = \frac{n}{1/x_1 + 1/x_2 + ...1/x_n} = \text{Reciprocal } \frac{\sum 1/x}{n}$$

Where

n is the total number of observations.

The main **advantage** of the harmonic mean is that it is based on all observations in a distribution and is amenable to further algebraic treatment. When we desire to give greater weight to smaller observations and less weight to the larger observations, then the use of harmonic mean will be more suitable. As against these advantages, there are certain limitations of the harmonic mean. First, it is difficult to understand as well as difficult to compute. Second, it cannot be calculated if any of the observations is zero or negative. Third, it is only a summary figure, which may not be an actual observation in the distribution.

It is worth noting that the harmonic mean is always lower than the geometric mean, which is lower than the arithmetic mean. This is because the harmonic mean assigns lesser importance to higher values. Since the harmonic mean is based on reciprocals, it becomes clear that as

reciprocals of higher values are lower than those of lower values, it is a lower average than the arithmetic mean as well as the geometric mean.

QUADRATIC MEAN

We have seen earlier that the geometric mean is the antilogarithm of the arithmetic mean of the logarithms, and the harmonic mean is the reciprocal of the arithmetic mean of the reciprocals. Likewise, the quadratic mean (Q) is the square root of the arithmetic mean of the squares. Symbolically,

$$Q = \sqrt{\frac{x_1^2 + x_2^2 + \ldots\ldots + x_n^2}{n}}$$

Instead of using original values, the quadratic mean can be used while averaging deviations when the standard deviation is to be calculated. This will be used in the next chapter on dispersion.

Relative Position of Different Means

The relative position of different means will always be:

Q> x >G>H provided that all the individual observations in a series are positive and all of them are not the same.

Composite Average or Average of Means

Sometimes, we may have to calculate an average of several averages. In such cases, we should use the same method of averaging that was employed in calculating the original averages. Thus, we should calculate the arithmetic mean of several values of *x,* the geometric mean of several values of GM, and the harmonic mean of several values of HM. It will be wrong if we use some other average in averaging of means.

Lecture III

Dispersion and Skewness

INTRODUCTION

In the previous chapter, we have explained the measures of central tendency. It may be noted that these measures do not indicate the extent of dispersion or variability in a distribution. The dispersion or variability provides us one more step in increasing our understanding of the pattern of the data. Further, a high degree of uniformity (i.e. low degree of dispersion) is a desirable quality. If in a business there is a high degree of variability in the raw material, then it could not find mass production economical.

Suppose an investor is looking for a suitable equity share for investment. While examining the movement of share prices, he should avoid those shares that are highly fluctuating-having sometimes very high prices and at other times going very low.

Such extreme fluctuations mean that there is a high risk in the investment in shares. The investor should, therefore, prefer those shares where risk is not so high.

MEANING AND DEFINITIONS OF DISPERSION

The various measures of central value give us one single figure that represents the entire data. But the average alone cannot adequately describe a set of observations, unless all the observations are the same. It is necessary to describe the variability or dispersion of the observations. In two or more distributions the central value may be the same but still there can be wide disparities in the formation of distribution.

Measures of dispersion help us in studying this important characteristic of a distribution.

Some important definitions of dispersion are given below:

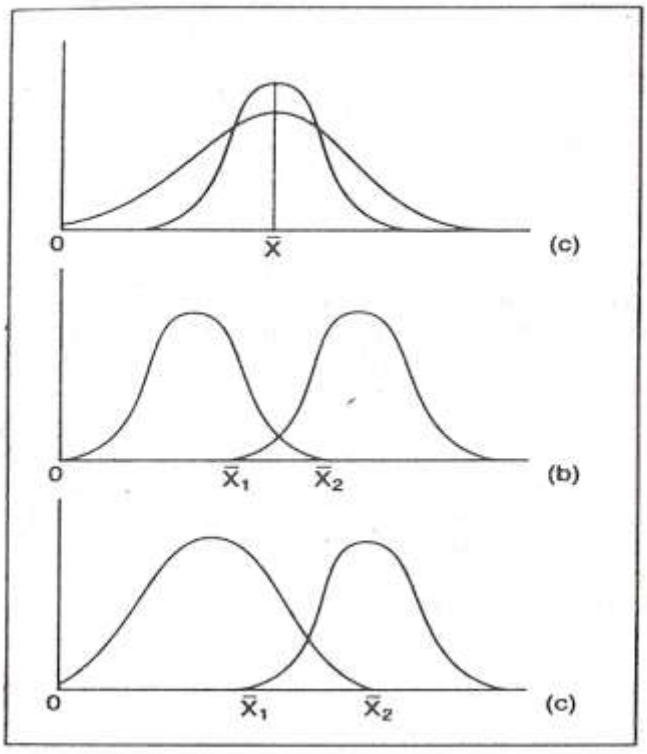
1. "Dispersion is the measure of the variation of the items." -A.L. Bowley
2. "The degree to which numerical data tend to spread about an average value is called the variation of dispersion of the data." -Spiegel
3. Dispersion or spread is the degree of the scatter or variation of the variable about a central value." -Brooks & Dick
4. "The measurement of the scatterness of the mass of figures in a series about an average is called measure of variation or dispersion." - Simpson & Kajka

It is clear from above that dispersion (also known as scatter, spread or variation) measures the extent to which the items vary from some central value. Since measures of dispersion give an

average of the differences of various items from an average, they are also called averages of the second order. An average is more meaningful when it is examined in the light of dispersion. For example, if the average wage of the workers of factory A is Rs. 3885 and that of factory B Rs. 3900, we cannot necessarily conclude that the workers of factory B are better off because in factory B there may be much greater dispersion in the distribution of wages. The study of dispersion is of great significance in practice as could well be appreciated from the following example:

	Series A	**Series B**	**Series C**
	100	100	1
	100	105	489
	100	102	2
	100	103	3
	100	90	5
Total	500	500	500
$\bar{x}$	100	100	100

Since arithmetic mean is the same in all three series, one is likely to conclude that these series are alike in nature. But a close examination shall reveal that distributions differ widely from one another.



In series A, (In Box-3.1) each and every item is perfectly represented by the arithmetic mean or in other words none of the items of series A deviates from the arithmetic mean and hence there is no dispersion. In series B, only one item is perfectly represented by the arithmetic mean and the other items vary but the variation is very small as compared to series C. In series C. not a single item is represented by the arithmetic mean and the items vary widely from one another. In series C, dispersion is much greater compared to series B. Similarly, we may have two groups of labourers with the same mean salary and yet their distributions may differ widely.

The mean salary may not be so important a characteristic as the variation of the items from the mean. To the student of social affairs the mean income is not so vitally important as to know how this income is distributed. Are a large number receiving the mean income or are there a few with enormous incomes and millions with incomes far below the mean? The three figures given in Box 3.1 represent frequency distributions with some of the characteristics. The two curves in diagram (a) represent two distractions with the same mean X , but with different dispersions. The two curves in (b) represent two distributions with the same dispersion but with unequal means X l and X 2, (c) represents two distributions with unequal dispersion. The measures of central tendency are, therefore insufficient. They must be supported and supplemented with other measures.

In the present chapter, we shall be especially concerned with the measures of variability or spread or dispersion. A measure of variation or dispersion is one that measures the extent to which there are differences between individual observation and some central or average value. In measuring variation we shall be interested in the amount of the variation or its degree but not in the direction. For example, a measure of 6 inches below the mean has just as much dispersion as a measure of six inches above the mean.

Literally meaning of dispersion is ‘scatteredness’. Average or the measures of central tendency gives us an idea of the concentration of the observations about the central part of the distribution. If we know the average alone, we cannot form a complete idea about the distribution. But with the help of dispersion, we have an idea about homogeneity or heterogeneity of the distribution.

SIGNIFICANCE AND PROPERTIES OF MEASURING VARIATION

Measures of variation are needed for four basic purposes:

1. Measures of variation point out as to how far an average is representative of the mass. When dispersion is small, the average is a typical value in the sense that it closely represents the individual value and it is reliable in the sense that it is a good estimate of the average in the corresponding universe. On the other hand, when dispersion is large, the average is not so typical, and unless the sample is very large, the average may be quite unreliable.

2. Another purpose of measuring dispersion is to determine nature and cause of variation in order to control the variation itself. In matters of health variations in body temperature, pulse beat and blood pressure are the basic guides to diagnosis. Prescribed treatment is designed to control their variation. In industrial production efficient operation requires control of quality variation the causes of which are sought through inspection is basic to the control of causes of variation. In social sciences a special problem requiring the measurement of variability is the measurement of "inequality" of the distribution of income or wealth etc.

3. Measures of dispersion enable a comparison to be made of two or more series with regard to their variability. The study of variation may also be looked upon as a means of determining uniformity of consistency. A high degree of variation would mean little uniformity or consistency whereas a low degree of variation would mean great uniformity or consistency.

4. Many powerful analytical tools in statistics such as correlation analysis. The testing of hypothesis, analysis of variance, the statistical quality control, regression analysis is based on measures of variation of one kind or another.

A good measure of dispersion should possess the following properties:

1. It should be simple to understand.
2. It should be easy to compute.
3. It should be rigidly defined.
4. It should be based on each and every item of the distribution.
5. It should be amenable to further algebraic treatment.
6. It should have sampling stability.
7. Extreme items should not unduly affect it.

MEAURES OF DISPERSION

There are five measures of dispersion: Range, Inter-quartile range or Quartile Deviation, Mean deviation, Standard Deviation, and Lorenz curve. Among them, the first four are mathematical methods and the last one is the graphical method. These are discussed in the ensuing paragraphs with suitable examples.

(i) RANGE

The simplest measure of dispersion is the range, which is the difference between the maximum value and the minimum value of data.
Example: Find the range for the following three sets of data:

Set 1:	05	15	15	05	15	05	15	15	15	15
Set 2:	8	7	15	11	12	5	13	11	15	9
Set 3:	5	5	5	5	5	5	5	5	5	5

Solution: In each of these three sets, the highest number is 15 and the lowest number is 5. Since the range is the difference between the maximum value and the minimum value of the data, it is 10 in each case. But the range fails to give any idea about the dispersal or spread of the series between the highest and the lowest value. This becomes evident from the above data.

In a frequency distribution, range is calculated by taking the difference between the upper limit of the highest class and the lower limit of the lowest class.

Example: Find the range for the following frequency distribution:

Size of Item	**Frequency**
20- 40	7
40- 60	11
60- 80	30
80-100	17
100-120	5
Total	**70**

Solution: Here, the upper limit of the highest class is 120 and the lower limit of the lowest class is 20. Hence, the range is 120 - 20 = 100. Note that the range is not influenced by the frequencies. Symbolically, the range is calculated b the formula L - S, where L is the largest value and S is the smallest value in a distribution. The coefficient of range is calculated by the formula: (L-S)/ (L+S). This is the relative measure. The coefficient of the range in respect of the earlier example having three sets of data is: 0.5.The coefficient of range is more appropriate for purposes of comparison as will be evident from the following example:

Example: Calculate the coefficient of range separately for the two sets of data given below:

Set 1	8	10	20	9	15	10	13	28
Set 2	30	35	42	50	32	49	39	33

Solution: It can be seen that the range in both the sets of data is the same:

Set 1 28 - 8 = 20

Set 2 50 - 30 = 20

Coefficient of range in Set 1 is:

$$\frac{28 - 8}{28+8} = 0.55$$

Coefficient of range in set 2 is:

$$\frac{50 - 30}{50 + 30} = 0.25$$

LIMITATIONS OF RANGE

There are some limitations of range, which are as follows:

1. It is based only on two items and does not cover all the items in a distribution.
2. It is subject to wide fluctuations from sample to sample based on the same population.
3. It fails to give any idea about the pattern of distribution. This was evident from the data given in Examples 1 and 3.
4. Finally, in the case of open-ended distributions, it is not possible to compute the range.

Despite these limitations of the range, it is mainly used in situations where one wants to quickly have some idea of the variability or' a set of data. When the sample size is very small, the range is considered quite adequate measure of the variability. Thus, it is widely used in quality control where a continuous check on the variability of raw materials or finished products is needed.

The range is also a suitable measure in weather forecast. The meteorological department uses the range by giving the maximum and the minimum temperatures. This information is quite useful to the common man, as he can know the extent of possible variation in the temperature on a particular day.

INTERQUARTILE RANGE OR QUARTILE DEVIATION

The interquartile range or the quartile deviation is a better measure of variation in a distribution than the range. Here, avoiding the 25 percent of the distribution at both the ends uses the middle 50 percent of the distribution. In other words, the interquartile range denotes the difference between the third quartile and the first quartile.

Symbolically, interquartile range = $Q_3 - Q_I$

Many times the interquartile range is reduced in the form of semi-interquartile range or quartile deviation as shown below:

Semi interquartile range or Quartile deviation = $(Q_3 - Q_I)/2$

When quartile deviation is small, it means that there is a small deviation in the central 50 percent items. In contrast, if the quartile deviation is high, it shows that the central 50 percent items have a large variation. It may be noted that in a symmetrical distribution, the two quartiles, that is, Q3 and QI are equidistant from the median.

Symbolically,

$$M-Q_l = Q_3-M$$

However, this is seldom the case as most of the business and economic data are asymmetrical. But, one can assume that approximately 50 percent of the observations are contained in the interquartile range. It may be noted that interquartile range or the quartile deviation is an absolute measure of dispersion. It can be changed into a relative measure of dispersion as follows:

$$\text{Coefficient of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

The computation of a quartile deviation is very simple, involving the computation of upper and lower quartiles. As the computation of the two quartiles has already been explained in the preceding chapter, it is not attempted here.

MERITS OF QUARTILE DEVIATION

The following merits are entertained by quartile deviation:
1. As compared to range, it is considered a superior measure of dispersion.
2. In the case of open-ended distribution, it is quite suitable.
3. Since it is not influenced by the extreme values in a distribution, it is particularly suitable in highly skewed or erratic distributions.

LIMITATIONS OF QUARTILE DEVIATION
1. Like the range, it fails to cover all the items in a distribution.
2. It is not amenable to mathematical manipulation.
3. It varies widely from sample to sample based on the same population.
4. Since it is a positional average, it is not considered as a measure of dispersion.

It merely shows a distance on scale and not a scatter around an average.
In view of the above-mentioned limitations, the interquartile range or the quartile deviation has a limited practical utility.

MEAN DEVIATION

The mean deviation is also known as the average deviation. As the name implies, it is the average of absolute amounts by which the individual items deviate from the mean. Since the positive deviations from the mean are equal to the negative deviations, while computing the mean deviation, we ignore positive and negative signs.

Symbolically,

$$MD = \frac{\Sigma | x |}{n}$$

where

MD = mean deviation, |x| = deviation of an item from the mean ignoring positive and negative signs, *n* = the total number of observations.

Example:

Size of Item	**Frequency**
2-4	20
4-6	40
6-8	30
8-10	10

Solution:

Size of Item	Mid-points (m)	Frequency (f)	fm	d	f/d/
2-4	3	20	60	-2.6	52
4-6	5	40	200	-0.6	24
6-8	7	30	210	-1.4	42
8-10	9	10	90	3.4	34
	Total	**100**	**500**		**152**

$$\bar{x} = \frac{\sum fm}{n} = \frac{560}{100} = 5.6$$

$$\text{MD } \bar{x} = \frac{\sum f|d|}{n} = \frac{152}{100} = 1.52$$

MERITS OF MEAN DEVIATION

1. A major advantage of mean deviation is that it is simple to understand and easy to calculate.
2. It takes into consideration each and every item in the distribution. As a result, a change in the value of any item will have its effect on the magnitude of mean deviation.
3. The values of extreme items have less effect on the value of the mean deviation.
4. As deviations are taken from a central value, it is possible to have meaningful comparisons of the formation of different distributions.

LIMITATIONS OF MEAN DEVIATION

1. It is not capable of further algebraic treatment.

2. At times it may fail to give accurate results. The mean deviation gives best results when deviations are taken from the median instead of from the mean. But in a series, which has wide variations in the items, median is not a satisfactory measure.
3. Strictly on mathematical considerations, the method is wrong as it ignores the algebraic signs when the deviations are taken from the mean.

In view of these limitations, it is seldom used in business studies. A better measure known as the standard deviation is more frequently used.

STANDARD DEVIATION

The standard deviation is similar to the mean deviation in that here too the deviations are measured from the mean. At the same time, the standard deviation is preferred to the mean deviation or the quartile deviation or the range because it has desirable mathematical properties.

Before defining the concept of the standard deviation, we introduce another concept viz. variance.

Example:

X	X-μ	$(X-\mu)^2$
20	20-18=12	4
15	15-18= -3	9
19	19-18 = 1	1
24	24-18 = 6	36
16	16-18 = -2	4
14	14-18 = -4	16
108	**Total**	**70**

Solution:

$$\text{Mean} = \frac{108}{6} = 18$$

The second column shows the deviations from the mean. The third or the last column shows the squared deviations, the sum of which is 70. The arithmetic mean of the squared deviations is:

$$\frac{\Sigma (x-\mu)^2}{N}$$

$$= 70/6 = 11.67 \text{ approx.}$$

This mean of the squared deviations is known as the variance. It may be noted that this variance is described by different terms that are used interchangeably: the variance of the distribution X; the variance of the distribution; and just simply, the variance.

Symbolically, Var (X) = $\frac{\Sigma (x-\mu)^2}{N}$

It is also written as $\sigma^2 = \frac{\Sigma (x-\mu)^2}{N}$

Where σ^2 (called sigma squared) is used to denote the variance.

Although the variance is a measure of dispersion, the unit of its measurement is (points). If a distribution relates to income of families then the variance is $(N)^2$ and not naira. Similarly, if another distribution pertains to marks of students, then the unit of variance is (marks)2. To overcome this inadequacy, the square root of variance is taken, which yields a better measure of dispersion known as the standard deviation.

Taking our earlier example of individual observations, we take the square root of the variance

SD or $\sigma = \sqrt{\text{var}\,iance} = \sqrt{11.67}$ = 3.42 points

Symbolically, $\sigma = \sqrt{\frac{\sum (x-\mu)^2}{N}}$

In applied Statistics, the standard deviation is more frequently used than the variance.
This can also be written as:

$$\sigma = \sqrt{\frac{\sum x_i^2 - \frac{(\sum x_i)^3}{N}}{N}}$$

We use this formula to calculate the standard deviation from the individual observations given earlier.

USES OF THE STANDARD DEVIATION

The standard deviation is a frequently used measure of dispersion. It enables us to determine as to how far individual items in a distribution deviate from its mean. In a symmetrical, bell-shaped curve:

(i) About 68 percent of the values in the population fall within: + 1 standard deviation from the mean.
(ii) About 95 percent of the values will fall within +2 standard deviations from the mean.
(iii) About 99 percent of the values will fall within + 3 standard deviations from the mean.

The standard deviation is an absolute measure of dispersion as it measures variation in the same units as the original data. As such, it cannot be a suitable measure while comparing two or more distributions. For this purpose, we should use a relative measure of dispersion. One such measure of relative dispersion is the coefficient of variation, which relates the standard deviation and the mean such that the standard deviation is expressed as a percentage of mean. Thus, the specific unit in which the standard deviation is measured is done away with and the new unit becomes percent.

Symbolically, CV (coefficient of variation) = $\frac{\sigma}{\mu} x100$

Example: In a small business firm, two typists are employed-typist A and typist B. Typist A types out, on an average, 30 pages per day with a standard deviation of 6. Typist B, on an average, types out 45 pages with a standard deviation of 10. Which typist shows greater consistency in his output?

Solution:

Coefficient of variation for A = $\frac{\sigma}{\mu} x100$

Or A = $\frac{6}{30} x100$

Or 20%

Coefficient of variation for B = $\frac{\sigma}{\mu} x100$

Or B = $\frac{10}{45} x100$

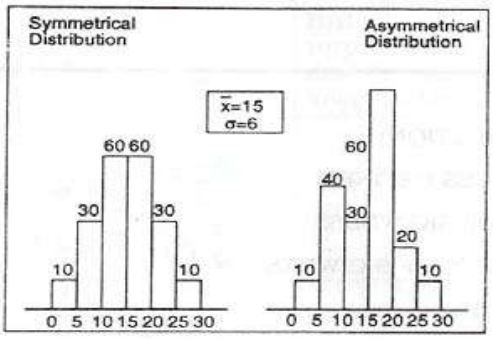
Or 22.2%

These calculations clearly indicate that although typist B types out more pages, there is a greater variation in his output as compared to that of typist A. We can say this in a different way: Though typist A's daily output is much less, he is more consistent than typist B. The

usefulness of the coefficient of variation becomes clear in comparing two groups of data having different means, as has been the case in the above example.

SKEWNESS: MEANING AND DEFINITIONS

In the above paragraphs, we have discussed frequency distributions in detail. It may be repeated here that frequency distributions differ in three ways: Average value, Variability or dispersion, and Shape. Since the first two, that is, average value and variability or dispersion have already been discussed in previous chapters, here our main spotlight will be on the shape of frequency distribution. Generally, there are two comparable characteristics called skewness and kurtosis that help us to understand a distribution. Two distributions may have the same mean and standard deviation but may differ widely in their overall appearance as can be seen from the following:



In both these distributions the value of mean and standard deviation is the same
(X = 15, σ = 5). But it does not imply that the distributions are alike in nature.
The distribution on the left-hand side is a symmetrical one whereas the distribution on the right-hand side is symmetrical or skewed. Measures of skewness help us to distinguish between different types of distributions.

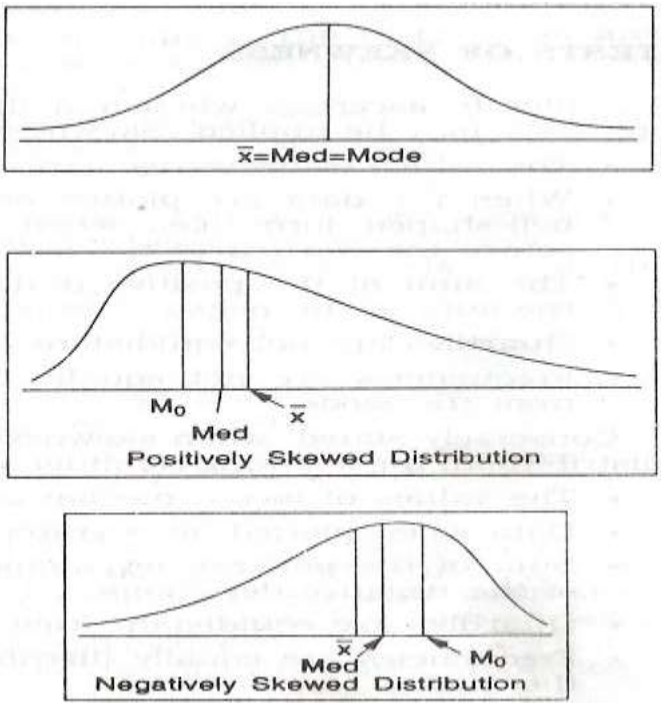
Some important definitions of skewness are as follows:

1. "When a series is not symmetrical it is said to be asymmetrical or skewed." -Croxton & Cowden.
2. "Skewness refers to the asymmetry or lack of symmetry in the shape of a frequency distribution." -Morris Hamburg.
3. "Measures of skewness tell us the direction and the extent of skewness. In symmetrical distribution the mean, median and mode are identical. The more the mean moves away from the mode, the larger the asymmetry or skewness." -Simpson & Kalka

4. "A distribution is said to be 'skewed' when the mean and the median fall at different points in the distribution, and the balance (or centre of gravity) is shifted to one side or the other-to left or right." -Garrett

The above definitions show that the term 'skewness' refers to lack of symmetry" i.e., when a distribution is not symmetrical (or is asymmetrical) it is called a skewed distribution.

The concept of skewness will be clear from the following three diagrams showing a symmetrical distribution. a positively skewed distribution and a negatively skewed distribution.



1. Symmetrical Distribution. It is clear from the diagram (a) that in a symmetrical distribution the values of mean, median and mode coincide. The spread of the frequencies is the same on both sides of the centre point of the curve.

2. Asymmetrical Distribution. A distribution, which is not symmetrical, is called a skewed distribution and such a distribution could either be positively skewed or negatively skewed as would be clear from the diagrams (b) and (c).

3. Positively Skewed Distribution. In the positively skewed distribution the value of the mean is maximum and that of mode least-the median lies in between the two as is clear from the diagram (b).

4. Negatively Skewed Distribution. The following is the shape of negatively skewed distribution. In a negatively skewed distribution the value of mode is maximum and that of

mean least-the median lies in between the two. In the positively skewed distribution the frequencies are spread out over a greater range of values on the high-value end of the curve (the right-hand side) than they are on the low-value end. In the negatively skewed distribution the position is reversed, i.e. the excess tail is on the left-hand side. It should be noted that in moderately symmetrical distributions the interval between the mean and the median is approximately one-third of the interval between the mean and the mode. It is this relationship, which provides a means of measuring the degree of skewness.

TESTS OF SKEWNESS

In order to ascertain whether a distribution is skewed or not the following tests may be applied. Skewness is present if:

1. The values of mean, median and mode do not coincide.
2. When the data are plotted on a graph they do not give the normal bell-shaped form i.e. when cut along a vertical line through the centre the two halves are not equal.
3. The sum of the positive deviations from the median is not equal to the sum of the negative deviations.
4. Quartiles are not equidistant from the median.
5. Frequencies are not equally distributed at points of equal deviation fromthe mode.

On the contrary, when skewness is absent, i.e. in case of a symmetrical distribution, the following conditions are satisfied:

1. The values of mean, median and mode coincide.
2. Data when plotted on a graph give the normal bell-shaped form.
3. Sum of the positive deviations from the median is equal to the sum of the negative deviations.
4. Quartiles are equidistant from the median.
5. Frequencies are equally distributed at points of equal deviations from the mode.

MEASURES OF SKEWNESS

There are four measures of skewness, each divided into absolute and relative measures. The relative measure is known as the coefficient of skewness and is more frequently used than the absolute measure of skewness. Further, when a comparison between two or more distributions is involved, it is the relative measure of skewness, which is used. The measures of skewness are:

(i) Karl Pearson's measure, (ii) Bowley’s measure, (iii) Kelly’s measure, and (iv) Moment’s measure. These measures are discussed briefly below:

(i) KARL PEARON'S MEASURE

The formula for measuring skewness as given by Karl Pearson is as follows:

Skewness = Mean – Mode

$$\text{Coefficient of skewness} = \frac{\text{Mean} - \text{Mode}}{\text{Standard Deviation}}$$

In case the mode is indeterminate, the coefficient of skewness is:

$$Sk_p = \frac{\text{Mean} - (3\text{Median} - 2\text{Mean})}{\text{Standard Deviation}}$$

$$Sk_p = \frac{3(3\text{Mean} - \text{Median})}{\text{Standard Deviation}}$$

Now this formula is equal to the earlier one.

Or 3 Mean - 3 Median = Mean - Mode
Or Mode = Mean - 3 Mean + 3 Median
Or Mode = 3 Median - 2 Mean

The direction of skewness is determined by ascertaining whether the mean is greater than the mode or less than the mode. If it is greater than the mode, then skewness is positive. But when the mean is less than the mode, it is negative. The difference between the mean and mode indicates the extent of departure from symmetry. It is measured in standard deviation units, which provide a measure independent of the unit of measurement. It may be recalled that this observation was made in the preceding chapter while discussing standard deviation. The value of coefficient of skewness is zero, when the distribution is symmetrical. Normally, this coefficient of skewness lies between +1. If the mean is greater than the mode, then the coefficient of skewness will be positive, otherwise negative.

(ii) Bowley's Measure

Bowley developed a measure of skewness, which is based on quartile values. The formula for measuring skewness is:

$$\text{Skewness} = \frac{Q1+Q3 - 2M}{Q3 - Q1}$$

Where *Q3* and *Q1* are upper and lower quartiles and *M* is the median. The value of this skewness varies between +1. In the case of open-ended distribution as well as where extreme values are found in the series, this measure is particularly useful. In a symmetrical distribution,

skewness is zero. This means that *Q3* and *Q1* are positioned equidistantly from *Q2* that is, the median. In symbols, *Q3 - Q2 = Q2 – Q1'* In contrast, when the distribution is skewed, then *Q3 - Q2* will be different from *Q2 – Q1'* When *Q3 - Q2* exceeds *Q2 – Q1'* then skewness is positive. As against this; when *Q3 - Q2* is less than *Q2 – Q1'* then skewness is negative. Bowley's measure of skewness can- be written as:

Skewness = *(Q3 - Q2) - (Q2 – Q1* or *Q3 - Q2 - Q2 + Q1*

Or *Q3 + Q1 - 2Q2 (2Q2* is *2M)*

However, this is an absolute measure of skewness. As such, it cannot be used while comparing two distributions where the units of measurement are different.

(iii) Kelly's Measure

Kelly developed another measure of skewness, which is based on percentiles. The formula for measuring skewness is as follows:

$$\text{Coefficient of Skewness} = \frac{P_{90} - 2P_{50} + P_{10}}{P_{90} - P_{10}}$$

Or

$$= \frac{D_1 + D_9 + 2M}{D_9 - D_1}$$

Where *P* and *D* stand for percentile and decile respectively. In order to calculate the coefficient of skewness by this formula, we have to ascertain the values of 10th, 50^{th} and 90th percentiles. Somehow, this measure of skewness is seldom used. All the same, we give an example to show how it can be calculated.

MOMENTS

In mechanics, the term *moment* is used to denote the rotating effect of a force. In Statistics, it is used to indicate peculiarities of a frequency distribution. The utility of moments lies in the sense that they indicate different aspects of a given distribution.

Thus, by using moments, we can measure the central tendency of a series, dispersion or variability, skewness and the peakedness of the curve. The moments about the actual arithmetic mean are denoted by μ. The first four moments about mean or *central moments* are as follows:

First moment $\mu_1 = \frac{1}{N}\sum(x_1 - \bar{x})$

Second moment $\mu_2 = \frac{1}{N}\sum(x_1 - \bar{x})^2$

Third moment $\mu_3 = \frac{1}{N}\sum(x_1 - \bar{x})^3$

Fourth moment $\mu_4 = \frac{1}{N}\sum(x_1 - \bar{x})^4$

These moments are in relation to individual items. In the case of a frequency distribution, the first four moments will be:

First moment $\mu_1 = \frac{1}{N}\sum f(x_1 - \bar{x})$

Second moment $\mu_2 = \frac{1}{N}\sum f(x_1 - \bar{x})^2$

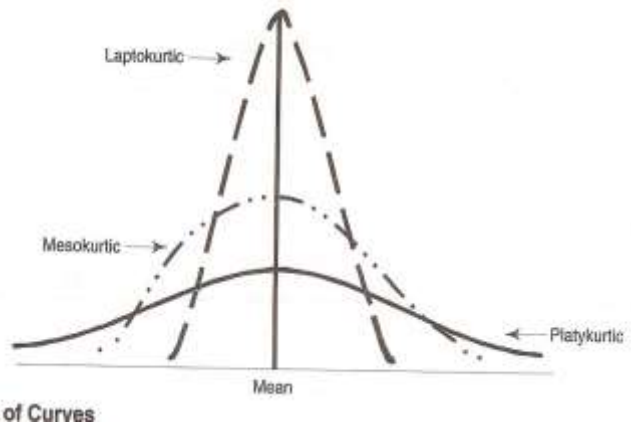
Third moment $\mu_3 = \frac{1}{N}\sum f(x_1 - \bar{x})^3$

Fourth moment $\mu_4 = \frac{1}{N}\sum f(x_1 - \bar{x})^4$

It may be noted that the first central moment is zero, that is, $\mu = 0$.
The second central moment is $\mu_2 = \sigma$, indicating the variance.
The third central moment μ_3 is used to measure skewness. The fourth central moment gives an idea about the Kurtosis.
Karl Pearson suggested another measure of skewness, which is based on the third and second central moments as given below:

KURTOSIS

Kurtosis is another measure of the shape of a frequency curve. It is a Greek word, which means bulginess. While skewness signifies the extent of asymmetry, kurtosis measures the degree of peakedness of a frequency distribution. Karl Pearson classified curves into three types on the basis of the shape of their peaks. These are mesokurtic, leptokurtic and platykurtic. These three types of curves are shown in figure below:



/pes of Curves

It will be seen from Fig. above that mesokurtic curve is neither too much flattened nor too much peaked. In fact, this is the frequency curve of a normal distribution. Leptokurtic curve is a more peaked than the normal curve. In contrast, platykurtic is a relatively flat curve. The coefficient of kurtosis as given by Karl Pearson is $\beta_2 = \mu_4 / \mu^2$. In case of a normal distribution, that is, mesokurtic curve, the value of $\beta_2 = 3$. If β_2 turn out to be > 3, the curve is called a leptokurtic curve and is more peaked than the normal curve.

Again, when $\beta_2 < 3$, the curve is called a platykurtic curve and is less peaked than the normal curve. The measure of kurtosis is very helpful in the selection of an appropriate average. For example, for normal distribution, mean is most appropriate; for a leptokurtic distribution, median is most appropriate; and for platykurtic distribution, the quartile range is most appropriate.